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FUNCTIONS DECREASING IN TRANSPOSITION
WITH APPLICATIONS TO SHOCK, DAMAGE, AND DOWN TIME.

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Functions Decreasing in Transposition with Applications to
Shock, Damage, and Down Time

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In this paper a large class of multivariate densities and frequency
functions, including the multivariate Poisson distribution and the com-
pound multivariate Poisson distribution, are shown to have the decreasing
in transposition property introduced by Hollander, Proschan, and Sethuraman
(1977, Ann. Statist. 5, 722-733). Applications relevant to reliability
are given, including applications to shock models, cumulation of damage,
and component down times.

1. INTRODUCTION AND PRELIMINARIES

In this paper we derive a basic theorem (Section 2, Theorem 2.1) show-
ing that a large class of multivariate probability densities and frequency
functions possess the decreasing in transposition (DT) property (Defini-
tion 1.3) introduced by Hollander, Proschan, and Sethuraman [3] (HPS[3]).
Of particular interest are the multivariate Poisson distribution and the
compound multivariate Poisson distribution treated in Section 3. Then,
in Section 4, we consider applications arising in reliability. These ap-
plications pertain to shock models, cummulation of damage, and component
down times. In the remainder of this section we give some preliminaries,
including definitions and theorems, which will be useful in the sequel.

HPS[3] define a partial ordering on n-dimensional Euclidean space (R^n) presented in Definition 1.2 below.

DEFINITION 1.1. A vector $\underline{x} = (x_1, \ldots, x_n)$ is said to be a simple transposition of a vector $\underline{x}'$ if $\underline{x}$ and $\underline{x}'$ agree in all but two coordinates, say i and j, $i < j$, $x_i < x_j$, $x_i' = x_j$, and $x_j' = x_i$; we write $\underline{x} \overset{t}{>} \underline{x}'$.

Thus $\underline{x}'$ is obtained from $\underline{x}$ by performing an inversion of a single pair of coordinates that occur in their natural order in $\underline{x}$.

DEFINITION 1.2. Let $\underline{x}$ and $\underline{x}'$ be two n-dimensional vectors such that there exists a finite number of vectors $\underline{x}^0, \underline{x}^1, \ldots, \underline{x}^k$ in R^n satisfying $\underline{x} = \underline{x}^0 \overset{t}{>} \underline{x}^1 \overset{t}{>} \ldots \overset{t}{>} \underline{x}^k = \underline{x}'$; i.e., x' is obtained from $\underline{x}$ by a finite number of simple transpositions. We say that $\underline{x}'$ is a transposition of $\underline{x}$.

HPS [3] then define a class of functions as follows:

DEFINITION 1.3. Let $\underline{\lambda} = (\lambda_1, \ldots, \lambda_n)$, where $\lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_n$ are n ordered values in R^1. We say that $f(\underline{\lambda}, \underline{x})$ is decreasing in transposition (DT) if:

(a) $f(\underline{\lambda}^\pi, \underline{x}^\pi) = f(\underline{\lambda}, \underline{x})$ for each permutation $\pi = (\pi_1, \ldots, \pi_n)$ of the indices $1, 2, \ldots, n$, where $\underline{\lambda}^\pi = (\lambda_{\pi_1}, \ldots, \lambda_{\pi_n})$ and $\underline{x}^\pi = (x_{\pi_1}, \ldots, x_{\pi_n})$.

(b) $\underline{x} \overset{t}{>} \underline{x}'$ implies that $f(\underline{\lambda}, \underline{x}) \geq f(\underline{\lambda}, \underline{x}')$.

The familiar concepts of majorization and Schur functions will be used in this paper, and for completeness we recall their definitions.

DEFINITION 1.4. Let $x_{[1]} \geq \ldots \geq x_{[n]}$ be a decreasing rearrangement of the coordinates of the vector $\underline{x}$. Let $\underline{x}$ and $\underline{y}$ satisfy:

$$\sum_{i=1}^{j} x_{[i]} \geq \sum_{i=1}^{j} y_{[i]}, \quad j = 1, \ldots, n - 1$$

$$\sum_{i=1}^{n} x_{[i]} = \sum_{i=1}^{n} y_{[i]}.$$

Then $\underline{x}$ is said to majorize $\underline{y}$ (we write $\underline{x} \overset{m}{\geq} \underline{y}$).

DEFINITION 1.5. A function f from R^n into R^1 is said to be Schur-convex (Schur-concave) if $\underline{x} \overset{m}{\geq} \underline{y}$ implies $f(\underline{x}) \geq (\leq) f(\underline{y})$.

HPS[3] derive basic properties of DT functions and show their relationship to other classes of functions. They show (HPS[3], Lemma 2.2) that when $f(\underline{\lambda}, \underline{x})$ is of the form $g(\underline{\lambda} - \underline{x})$, $f(\underline{\lambda}, \underline{x})$ is a DT function if and only if $g(\underline{y})$ is a Schur-concave function. They also show (HPS[3], Theorem 3.2) that the DT property is preserved under positive mixtures and (HPS[3], Theorem 3.6) that the DT property is preserved under products of positive DT functions. They also prove a "composition" theorem for DT functions (HPS[3], Theorem 3.3) and a "preservation" theorem (HPS[3], Theorem 3.7) for Schur-concave functions under an integral transform where the kernel is DT.

By restricting $g_2(\underline{y}, \underline{z})$ in the HPS "composition" theorem to be of the form $g(\underline{y} - \underline{z})$, where $g(\underline{w})$ is Schur-concave, we have the following.

THEOREM 1.5. Let $g_1(\underline{x}, \underline{y})$ be a DT function and $g(\underline{w})$ be a Schur-concave function such that

$$f(\underline{x}, \underline{z}) = \int g_1(\underline{x}, \underline{y}) g(\underline{y} - \underline{z}) \, d\mu(\underline{y})$$

is well-defined, where μ is a positive permutation invariant measure. Then $f(\underline{x}, \underline{z})$ is a DT function.

Theorem 1.5 generalizes the following result of Marshall and Olkin ([5], Theorem 2.1).

THEOREM 1.6. (Marshall-Olkin, [5]). Let $g_1(\underline{x})$ and $g_2(\underline{x})$ be Schur-concave functions such that

$$f(\underline{x}) = \int g_1(\underline{x} - \underline{y}) g_2(\underline{y}) d\mu(\underline{y})$$

is well-defined, where μ is as in Theorem 1.5. Then $f(\underline{x})$ is a Schur-concave function.

2. DT PROPERTY OF OVERLAPPING SUMS

In this section we state and prove our main theorem. Let, for $k = 2, 3, \ldots, n$,

$$\Lambda_k = \{\lambda: \ \lambda \text{ is a subset of size k from } \{1, \ldots, n\}\}. \quad (2.1)$$

THEOREM 2.1. Let $\underline{\ } = (X_1, \ldots, X_n)$, $\{X_\lambda, \lambda \in \Lambda_k\}$, $k = 2, \ldots, n$,

be independent collections of random variables. Let $\underline{X}$ have a DT density

function. Let the random variables in $\{X_\lambda, \lambda \in \Lambda_k\}$ be i.i.d. and have

a common log-concave density function g_k, $k = 2, \ldots, n$. Let, for   .

$i = 1, \ldots, n$,

$$Z_i = X_i + \sum_{k=2}^{n} \sum_{\lambda: \ \lambda \in \Lambda_k \ \text{and} \ i \in \lambda} X_\lambda \qquad . \qquad (2.2)$$

Then $\underline{Z} = (Z_1, \ldots, Z_n)$ has a DT density function.

REMARK 2.2. Note that the summands appearing in the $Z_1, \ldots, Z_n$ overlap

considerably. For example, X_{12} appears in the expressions for Z_1 and Z_2,

X_{123} appears in the expressions for Z_1, Z_2, and Z_3, etc. Thus the inheri-

tance of the DT property of $\underline{\ }$ from that of $\underline{X}$ is complicated by the over-

lapping of the X's.

To prove the main result, we shall find it helpful to have available

the following lemma:

LEMMA 2.3   Let $k \geq 2$. Let $\{X_\lambda, \lambda \in \Lambda_k\}$ be i.i.d. random variables

with a common log-concave density function g (with respect to the counting

measure on a lattice or the Lebesgue measure). Let

$$W_i = \sum_{\lambda: \ \lambda \in \Lambda_k \ \text{and} \ i \in \lambda} X_\lambda \qquad , \ i = 1, \ldots, n. \qquad (2.3)$$

Let $f(w_1, \ldots, w_n)$ be the density function of $\underline{W} = (w_1, \ldots, w_n)$. Then f

is a Schur-concave function.

Proof. We will now give a proof for the case where g and f are density functions with respect to counting measures. The proof for the case where g and f are density functions with respect to Lebesgue measures is similar and will be omitted.

Notice first that $W_1, \ldots, W_n$ are exchangeable and hence $f(w_1, \ldots, w_n)$ is permutation invariant. Fix $w_3, \ldots, w_n$ and define

$$A_{w_1, w_2} = \{W_1 = w_1, W_2 = w_2, W_3 = w_3, \ldots, W_n = w_n\}.$$

To show that f is Schur-concave, we must show that

$$P(A_{w_1, w_2}) \leq P(A_{w_1', w_2'}) \tag{2.4}$$

whenever

$$(w_1, w_2) \overset{m}{\geq} (w_1', w_2'),$$

i.e. whenever

$$0 \leq \beta \leq \alpha \leq w^* \tag{2.5}$$

where

$$w_1 = w^* + \alpha, \; w_2 = w^* - \alpha, \; w_1' = w^* + \beta, \; w_2' = w^* - \beta.$$

The following classes of subsets of $\{1, \ldots, n\}$, will help us to divide the elements of Λ into those that contain both 1 and 2, and those that do not contain either 1 or 2, and those that contain exactly one of 1 and 2. Define

$$V = \{v: \; v \text{ is a subset of size } (k - 2) \text{ from } \{3, \ldots, n\}\},$$

$$U = \{u: \; u \text{ is a subset of size } (k - 1) \text{ from } \{3, \ldots, n\}\},$$

and

$$S = \{s: \ s \text{ is a subset of size } k \text{ from } \{3, \ldots, n\}\}.$$

Any λ in Λ_k is of the form $12v$, $1u$, $2u$, or s. Fix numbers x_{12v}, $v \in V$, x_s, $s \in S$ and x_u, $u \in U$ and define the event B as follows:

$$B = \{X_{12v} = x_{12v}, v \in V, X_s \approx x_s, s \in S, X_{1u} + X_{2u} = 2x_u, u \in U\}. \quad (2.6)$$

Let

$$2Y_u = X_{1u} - X_{2u}, u \in U.$$

Then the density function of $\{Y_u, u \in U\}$ conditional on B is

$$\frac{\prod_u f(x_u + y_u) f(x_u - y_u)}{\sum_{y_u, u \in U} \prod_u [f(x_u + y_u) f(x_u - y_u)]}, \quad (2.7)$$

and the conditional probability of A_{w_1, w_2} given B is

$$P(A_{w_1, w_2} | B) = \begin{cases} \dfrac{\sum_{\sum y_u = \alpha} \prod_u f(x_u + y_u) f(x_u - y_u)}{\sum_{y_u, u \in U} \prod_u f(x_u + y_u) f(x_u - y_u)} \\ \quad \text{if } w_j = \sum_{v: \, j \in v} x_{12v} + \sum_{u: \, j \in u} x_u + \sum_{s: \, j \in s} x_s, \ j = 3, \ldots, n \\ \quad \text{and } w^* = \sum_v x_{12v} + \sum_u x_u \\ \\ 0 \qquad \text{otherwise.} \end{cases}$$

Now, for each x_u, $f(x_u + y_u) f(x_u - y_u)$ is symmetric and log-concave in y_u. The numerator in the expression for $P(A_{w_1, w_2} | B)$ is therefore a convolution

of symmetric log-concave densities and hence, from Theorem 1 of Karlin and Proschan [4], is symmetric and log-concave in α. Thus if (2.5) is satisfied, we obtain

$$P(A_{w_1,w_2} | B) \leq P(A_{w_1',w_2'} | B).$$

Inequality (2.4) now follows by unconditioning, and thus we have established that f is schur-concave. $||$

We now give the proof of the main theorem.

Proof of Theorem 2.1. Let

$$X_i^{(k)} = \sum_{\lambda:\ \lambda \in \Lambda_k \text{ and } i \in \lambda} X_\lambda \qquad , i = 1, \ldots n, k = 2, \ldots, n.$$

Let $\underline{X}^{(k)} = (X_1^{(k)}, \ldots, X_n^{(k)})$, $k = 2, \ldots, n$, and $\underline{X} = (X_1, \ldots, X_n)$. Then

$$\underline{Z} = \underline{X} + \underline{X}^{(2)} + \ldots + \underline{X}^{(n)}.$$

From Lemma 2.3, the density functions of $\underline{X}^{(2)}, \ldots, \underline{X}^{(n)}$ are all Schur-concave. Since the density function of $\underline{Z}$ is the convolution of a DT density and several Schur-concave densities, it follows from Theorems 1.6 and 1.7 that the density function of $\underline{Z}$ is DT. $||$

3. APPLICATIONS TO MULTIVARIATE POISSON DISTRIBUTIONS

In this section we present several applications of Theorem 2.1.

3.1. MULTIVARIATE POISSON. The multivariate Poisson distribution (cf. Teicher [5], Dwass and Teicher [1]) can be defined as follows. Let X_i be a Poisson random variable with mean m_i, $i = 1, \ldots, n$, X_λ be a Poisson random variable with mean m_λ, $\lambda \in \Lambda_k$, $k = 2, \ldots, n$, where Λ_k is

given by (2.1). All the Poisson random variables are assumed mutually independent. Then $(Z_1, \ldots, Z_n)$, as defined by (2.2), is said to have a joint multivariate Poisson distribution.

Now we specialize further to the case where

$$m_\lambda = m_k^*, \; \lambda \epsilon \Lambda_k, \; k = 2, \ldots, n. \tag{3.1}$$

That is, all X's with the same number (> 1) of subscripts, have a common mean. Then, from Theorem 2.1, we conclude that the corresponding frequency function is decreasing in transposition. That is, let $\underline{m} = (m_1, \ldots, m_n)$ and let $f(\underline{m}, \underline{z})$ denote the joint probability mass function of $Z_1, \ldots, Z_n$ (where the other means are held fixed as in (3.1)). Then when $m_1 \leq \ldots \leq m_n$, and $\underline{z} \overset{t}{\geq} \underline{z}'$, we have $f(\underline{m}, \underline{z}) \geq f(\underline{m}, \underline{z}')$.

3.2. COMPOUND MULTIVARIATE POISSON. The compound multivariate Poisson distribution described below generalizes the compound bivariate Poisson distribution considered by Holgate [2] in the context of certain ecological situations. Let $Z_1, Z_2, \ldots, Z_n$ denote the number of individuals of type 1, 2, ..., n, respectively, in a quadrat of land. We suppose that these individuals arise from independent clusters and assume that the number of clusters N is a Poisson random variable with parameter λ. In cluster j, there are

$$z_i^j = x_i^j + \sum_{k=2}^n \sum_{\lambda: \; \lambda \epsilon \Lambda_k \text{ and } i \epsilon \lambda} x_\lambda^i \quad ,$$

individuals of type i, i = 1, ..., n,

where X_i^j, X_λ^j, $\lambda \in \Lambda_k$, $k = 2, \ldots, n$, are independent Poisson random variables with parameters m_i, m_λ, $k = 2, \ldots, n$, respectively, and satisfying $m_\lambda = m_k^*$ for $\lambda \in \Lambda_k$, $k = 2, \ldots, n$, for each cluster j. We have already stated that the clusters are assumed to be independent. Thus $\underline{Z}^j = (Z_1^j, \ldots, Z_n^j)$ has a multivariate Poisson distribution. The vector $\underline{Z}$ of the number of individuals of the different types in the quadrat is given by

$$\underline{Z} = \underline{Z}^1 + \ldots + \underline{Z}^N$$

and is said to have a compound multivariate Poisson distribution. We now show that the density of $\underline{Z}$ is DT. Conditional on $N = N_0$, $\underline{Z}$ is the sum of N_0 i.i.d. multivariate Poisson vectors and therefore is multivariate Poisson from the infinite divisibility of the multivariate Poisson established by Dwass and Teicher [1]. Thus the conditional density of $\underline{Z}$, given $N = N_0$, is DT. By unconditioning, the density of $\underline{Z}$ is also DT. This proof also shows that the random number of clusters N, could be any random variable taking values on the positive integers.

4. APPLICATIONS TO RELIABILITY PROBLEMS

In this section we show sample applications of Theorem 2.1 in some commonly occurring reliability models.

4.1. NUMBER OF SHOCKS EXPERIENCED BY COMPONENTS OF SYSTEM. Consider a system of n components experiencing successive shocks of various types over time, each shock affecting one or more components. More speci-

fically, in a fixed interval of time, let X_i be the number of shocks affecting component i alone (i = 1, 2, ..., n), X_{ij} the number of shocks affecting components i and j simultaneously and no other components ($1 \leq i < j \leq n$), ..., $X_{12...n}$ the number of shocks affecting all n components simultaneously. Let (Z_1, ..., Z_n) be defined as in (2.2). Then Z_i denotes the total number of shocks experienced by component i, i = 1, ..., n. Assume that (X_1, ..., X_n), $\{X_\lambda, \lambda \in \Lambda_k\}$, k = 2, ..., n satisfy the conditions of Theorem 2.1. Then we conclude that the joint frequency function of (Z_1, ..., Z_n) is DT.

4.2. CUMULATION OF DAMAGE. We consider a model similar to that of 4.1. However, now we study the damage accumulated by each component as a result of the shocks simultaneously affecting the various subsets of components. Let D_i be the damage accumulated by component i as a result of shocks affecting component i alone (i = 1, ..., n), D_{ij} be the damage accumulated by components i and j as a result of shocks affecting components i and j simultaneously, but no other components ($1 \leq i < j \leq n$), ..., and $D_{12...n}$ be the damage accumulated by each of the components as a result of shocks affecting all components simultaneously. Assume that the joint distribution of $\underline{D} = (D_1, ..., D_n)$ has a DT density function, that D_λ, $\lambda \in \Lambda_k$ has a common log concave density function f_k, k = 2, ..., n. Assume further that $\underline{D}$; $\{D_\lambda, \lambda \in \Lambda_k\}$, k = 2, ..., n are mutually dependent.
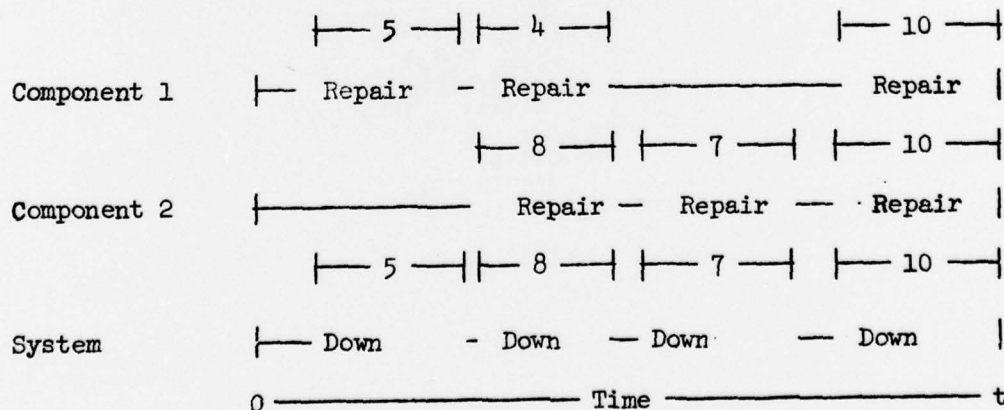
Define:

$$T_i = D_i + \sum_{k=2} \sum_{\lambda:\ \lambda\epsilon\Lambda_k,\, i\epsilon\lambda} D_\lambda \qquad ,\ i = 1,\ \ldots,\ n.$$

T_i represents the total damage to component i accruing from all shocks affecting component i (and possibly affecting other components simultaneously), i = 1, ..., n.

From Theorem 2.1 we conclude that the joint probability density of T_1, ..., T_n is DT.

4.3. COMPONENT DOWN TIMES. Suppose that an n-component system is subject to repair (replacement). If any component fails, the system fails and the repair (replacement) time for that component is recorded. Likewise, if any subset of components fails simultaneously, the system fails, and the time it takes to repair (replace) each of the components in that subset is recorded. For example, the operating record of a two-component system is exhibited in Fig. 4.1 below.

```
              |— 5 —| |— 4 —|                    |— 10 —|
Component 1   |— Repair  - Repair ———————————— Repair  |

                      |— 8 —| |— 7 —|  |— 10 —|
Component 2   |——————————— Repair — Repair — ·Repair  |

              |— 5 —| |— 8 —| |— 7 —|  |— 10 —|
System        |— Down    - Down  — Down    — Down      |

              0 ——————————————— Time ——————————————— t
```

4.1. Operating Record of a Two-Component System

Figure 4.1 indicates that a total of 4 repair (replacement) periods occurred during $[0, t]$: 1 period was due to the failure of component 1 alone, 1 period was due to the failure of component 2 alone, and 2 periods were due to the simultaneous failure of components 1 and 2.

Let R_i = time devoted to repair of component i due to shocks affecting component i alone $(1 \leq i \leq n)$, R_λ = time devoted to repair of each of components in λ due to shocks affecting components in λ simultaneously and no other components, for each $\lambda \in \Lambda_k$, $k = 2, \ldots, n$. Assume that the joint distribution of $R_1, \ldots, R_n$ has a DT density function, that R_λ has a common log concave density function f_k for $\lambda \in \Lambda_k$, $k = 2, \ldots, n$. Assume further that $\underline{R}$, $\{R_\lambda, \lambda \in \Lambda_k\}$, $k = 2, \ldots, n$ are mutually independent. Define

$$S_i = R_i + \sum_{k=2}^{n} \sum_{\lambda:\ \lambda \in \Lambda_k, i \in \lambda} R_\lambda \quad , i = 1, \ldots, n.$$

S_i represents the total time spent repairing (replacing) component i as a result of shocks affecting component i, $i = 1, \ldots, n$.

From Theorem 2.1, we conclude that the joint density function of $S_1, \ldots, S_n$ is DT.

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